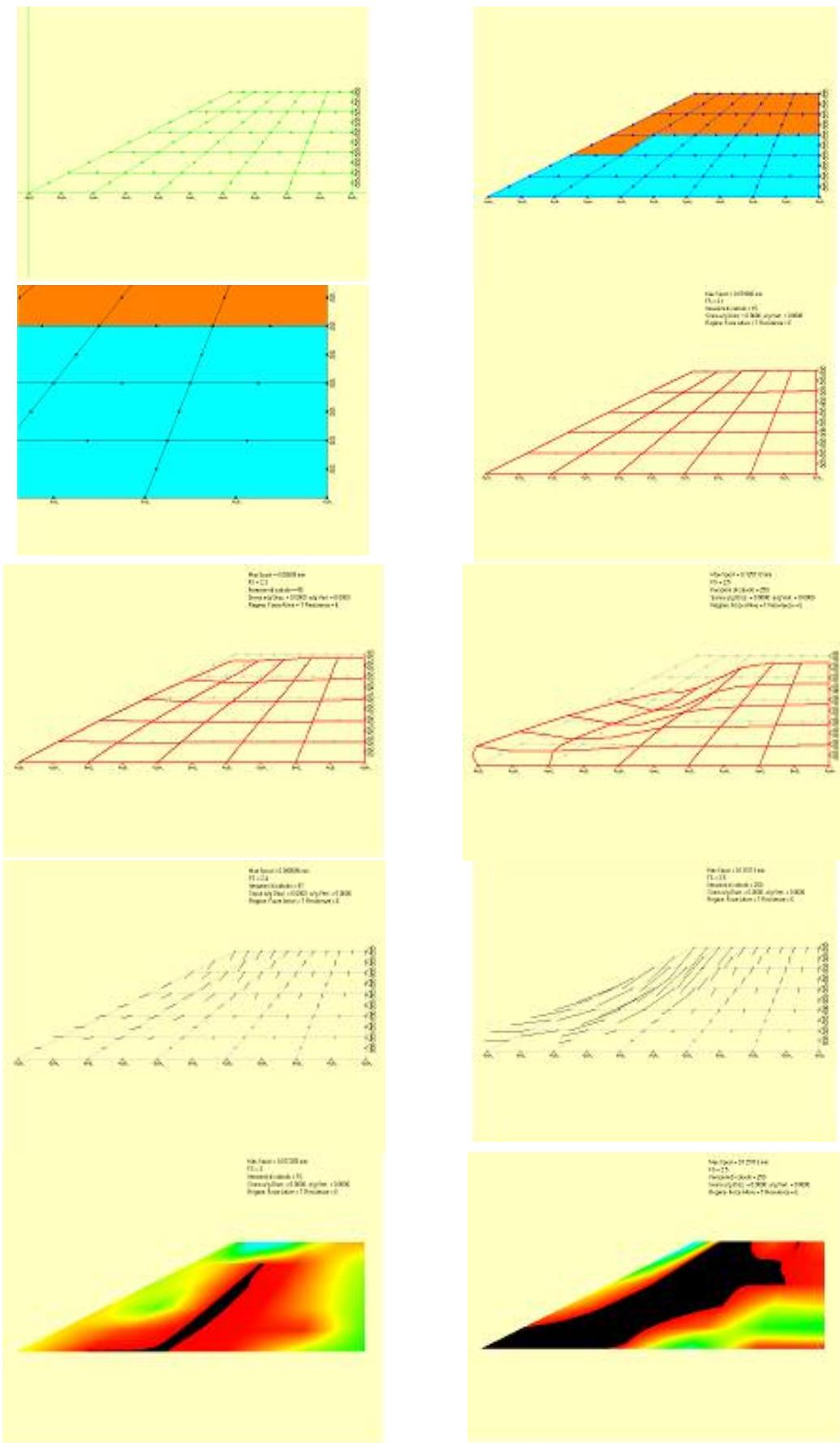


FEA – Slope – A Finite Element Analysis of Slopes

VALIDATION MANUAL



MANUALE DI VALIDAZIONE

La validazione del modello FEM viene eseguita in parte ripercorrendo alcune soluzioni pubblicate da fonti riconosciute e paragonando i risultati ottenuti, in parte mediante considerazioni basilari riguardo alla teoria geotecnica.

Tra le soluzioni pubblicate:

- pendio omogeneo coesivo-attributivo con falda: vedere T. W. Lambe – R.V. Whitman (1979): "Soil Mechanics" – Wiley, Example 24.3
- pendio non omogeneo su basamento roccioso rigido: vedere Navfac D.M. 7.01 (1986): "Soil Mechanics", Tab. 7.1-323
- pendio omogeneo coesivo: vedere Navfac D.M. 7.01 (1986): "Soil Mechanics", Tab. 7.1-319
- pendio omogeneo incoerente analizzato con algoritmo FEM: vedere Smith-Griffiths (2004) "Programming the Finite Element Method" – Wiley, Program 6.3

Queste soluzioni sono reperibili e riproducibili nei Files Validazione di FEA Slope.

Tra le soluzioni sulla teoria geotecnica:

- pendio omogeneo, materiale attritivo, con pendenza pari all'angolo di attrito $\phi = 30^\circ$
- pendio omogeneo, materiale attritivo, con pendenza pari all'angolo di attrito $\phi = 35^\circ$
- pendio omogeneo, materiale attritivo, con pendenza pari all'angolo di attrito $\phi = 40^\circ$

In tutti i tre casi il pendio collassa per valori di F_s compresi tra 1.000 e 1.005.

EXAMPLE LAMBE WHITMAN c- ϕ SOIL WITH GROUNDWATER

GEOTECHNICAL PARAMETERS

Natural Unit Weight	γ	=	19.64	KN/m ³
Submerged Unit Weight	γ_1	=	9.83	KN/m ³
Cohesion	C	=	4.31	KPa
Angle of Internal Friction	ϕ	=	32	°
Dilatancy	δ	=	0	°
Poisson's Ratio	ν	=	0.25	
Soil Modulus (Young's Modulus)	E	=	25.0	MPa

REFERENCE (PUBLISHED) SAFETY FACTOR	FEA SLOPE SAFETY FACTOR
1.27 - 1.29	1.26

FEA SLOPE MAX INTERACTIONS	=	500
FEA SLOPE MAXIMUM ABSOLUTE DEFLECTION	=	22.4 mm

► Example 24.3

Given. The slope, failure surface, flow net, and strength parameters in Fig. E24.3-1.

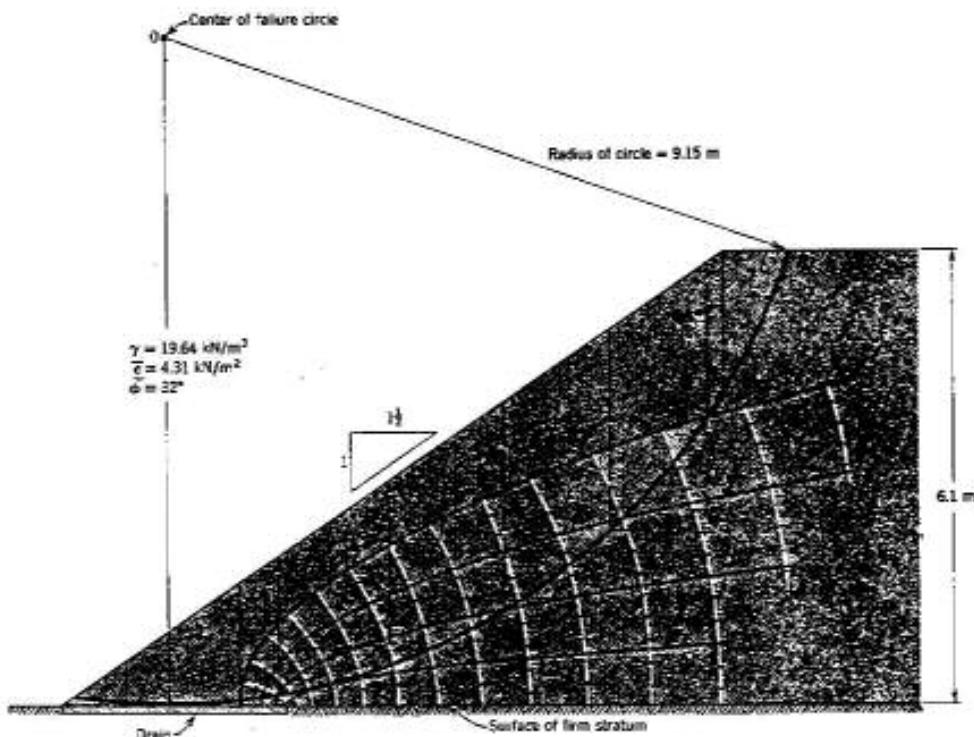


Fig. E24.3-1

Find. The safety factor.

Solution. The first step is to find the weight of the free body above the failure surface. This may be done conveniently by breaking the free body into a series of vertical slices as shown in the figure. Slices 2 to 6 are approximately trapezoids, and their weight can be computed by multiplying the unit weight of the soil times the width of the slice times the average height of the slice. Slices 1 and 7 may similarly be treated as triangles. The calculation of the resulting weight is given in Table E24.3.

The next step is to determine the resultant of the pore water pressures along the failure arc. Figure E24.3-2 illustrates the evaluation of the pore water force on the base of one slice: slice 4. The forces on the several slices is summed vectorially, giving the resultant force U . This force must act through the center of the failure circle.

The next step is to construct a force polygon. This is done as follows (refer to Fig. 24.9):

1. Lay off the line of actions of W and U and find their intersection (point A in Fig. E24.3-3).
2. Determine graphically the resultant Q of W and U . Q must act through point A .
3. Determine the line of action of R_c . The moment of the cohesive stresses about O is $cL_s r/F$ where L_s is the length of the failure arc. However, the resultant R_c is cL/F where L is the length of the chord of the failure arc, because components of c normal to the chord

Example 24.3 (continued)

Table E24.3

Slice	Width (m)	Average Height (m)	Weight (kN)	Moment about O (kN-m)
1	1.37	0.49	13.2	-9.0
2	0.98	1.28	24.6	12.1
3	0.55	1.77	19.1	24.0
4	1.52	2.26	67.5	154.6
5	1.52	2.74	81.8	311.7
6	1.52	2.84	84.8	452.0
6.4	0.18	2.56	67.4	455.6
7	0.98	2.04	7.2	54.1
			22.3	180.6
$\Sigma W = 387.9$				1635.7

Resultant lies $1635.7/387.9 = 4.2$ m to right of center of circle.

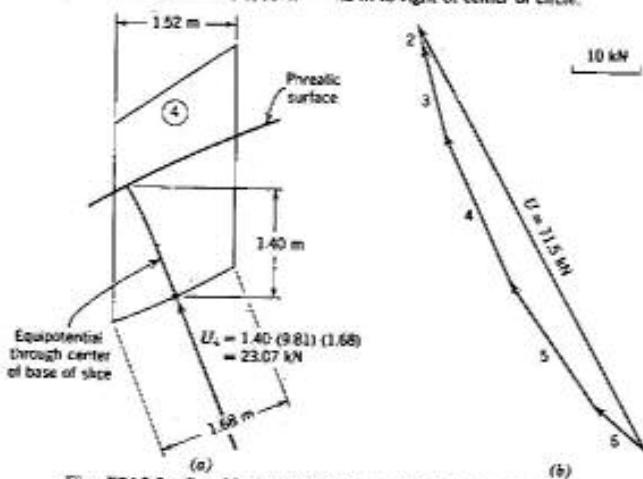


Fig. E24.3-2 Graphical solution for resultant pore water force.

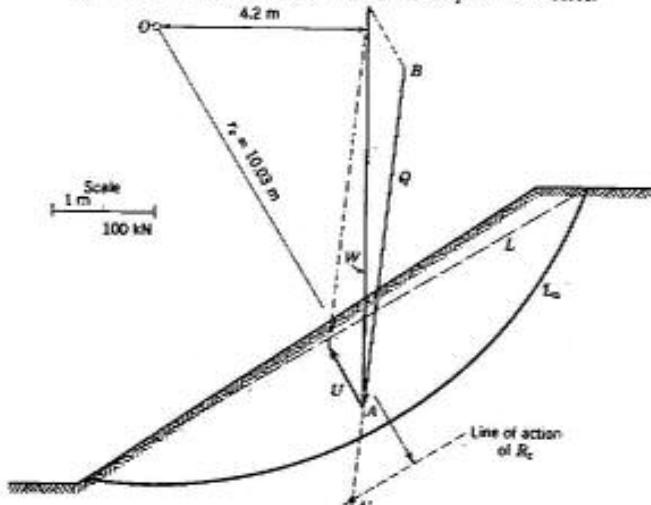


Fig. E24.3-3 Location of point A'.

Example 24.3 (continued)

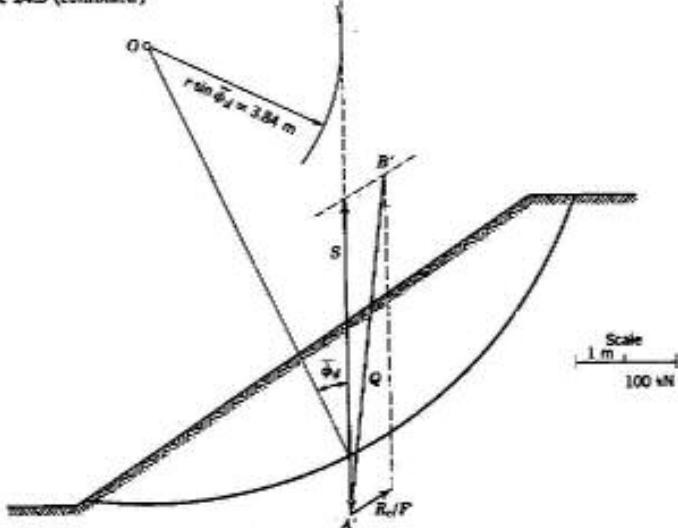


Fig. E24.3-4 Force equilibrium.

cancel and give no net force. Hence

$$r_s R_s = r_s \frac{EL}{F} = \frac{\delta L_{sf}}{F} \quad \text{or} \quad r_s = \frac{L_{sf}}{E} r$$

r_s is 10.03 m for this case.

4. Determine the location of point A' by the intersection of Q and R_c . The force S , the resultant of R and R_c , must act through A' (see Fig. E24.3-4). Assuming $r_s = r$, the line of action of S must make an angle ϕ_d with the radius through the intersection of S with the failure arc, where ϕ_d is given by

$$\tan \phi_d = \frac{\tan \phi}{F}$$

Thus S must pass tangent to a circle having a radius $r \sin \phi_d$. This is the *friction circle*.

5. Equilibrium is satisfied by a closed force polygon involving Q , S , and R_c/F . A trial and error procedure is necessary to find the solution. Several friction circles are assumed, thus permitting the polygon to be closed. For each assumed circle, two safety factors are obtained:

$$F_\phi = \frac{\tan \phi}{\tan \phi_d}$$

$$F_c = \frac{EL}{R_s}$$

The correct solution is that giving $F_\phi = F_c$. For example, for the solution in Fig. E24.3-4,

$$r \sin \phi_d = 3.84 \text{ m} \quad \phi_d = 25^\circ; \quad F_\phi = 1.34$$

$$\frac{R_s}{F} = 45.96 \text{ kN}, \quad LE = 11.59(4.31) = 49.93 \text{ kN}, \quad F_c = 1.09$$

The correct safety factor satisfying statics is $F = 1.27$ (see Fig. E24.3-5).

This is the answer for the given circle. Now other circles must be analyzed until the circle giving the smallest F is found. The circle given actually is the critical circle.

As discussed in the text, the foregoing solution with $r = r_s$ gives a lower bound. Figure E24.3-6 shows a trial solution based on the assumption that normal stress against the failure arc is concentrated at the two ends. Then S does not pass tangent to the friction circle; rather it acts as shown.

Example 24.3 (continued)

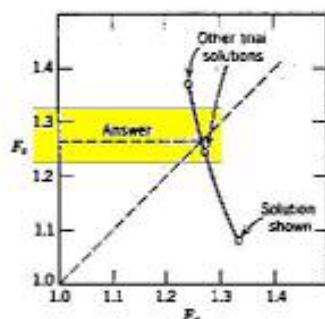
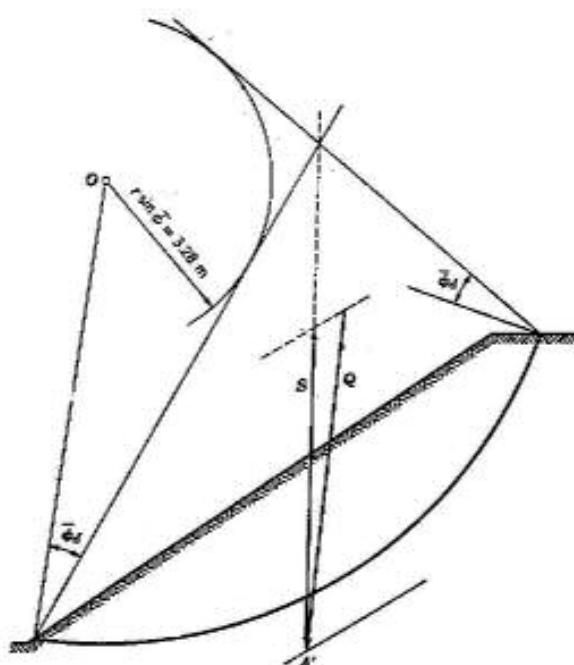


Fig. E24.3-5 Safety factors.

Fig. E24.3-6 Solution for upper bound. Assumed $\phi_d = 21^\circ$, $F_q = 1.63$, $R_s/F = 31.37 \text{ kN}$, $F_t = 49.93/31.37 = 1.59$. Further trials give $F = 1.61$.

which is broken into n vertical slices. If the slices are made so thin that the coordinates a_i (which determine the location of the resultants R_i along the segments of the failure arc) can be taken as zero, then there are $4n - 2$ unknowns versus $3n$ equations, or $n - 2$ extra unknowns. Breaking the mass up into a series of vertical slices does not remove the problem of statical indeterminacy. Hence in order to obtain values of the safety factor by using the method of slices, it is still necessary

to make assumptions to remove the extra unknowns. The value of safety factor computed thereby will, of course, depend on the reasonableness of the assumptions that have been made.

Usually assumptions are made regarding the forces that act against the sides of the slices. If the problem is to become statically determinate, exactly $n - 2$ assumptions must be made. A discussion of the best way in which to make these assumptions, and of the techniques

for solving the resulting system of simultaneous equations is beyond the scope of this text (see Morgenstern and Price, 1965; Whitman and Bailey, 1967). Careful analysis shows that there are severe limitations on the way in which these assumptions can be made: the shear forces on the side of the slices cannot exceed the shear resistance of the soil, and the side forces E_i should fall at a distance above the failure arc between one-third and one-half of the height of the slice. Hence, although a wide range of safety factors can be computed based on the assumptions, there is only a narrow range of safety factors corresponding to an intuitively reasonable distribution of stress along the failure arc and within the failure mass. For the slope in Example 24.3, this range is again from 1.30 to 1.36.

Use of a method of slices that takes full account of side forces and fully satisfies equilibrium requires use of a computer (see Whitman and Bailey, 1967). Even then there are considerable complexities involved in the use of such a method. This method can and should be used for advanced stages of slope stability studies, and is especially useful for the study of noncircular failure surfaces. For many problems, however, it is sufficient to use approximate methods, which do not fully satisfy the requirements of static equilibrium but have been found to give reasonably correct answers for most problems. Several such methods will now be described.

Features Common to All Approximate Methods

In all of these methods, the safety factor is defined in terms of moments about the center of the failure arc:

$$F = \frac{M_R}{M_D}$$

$$= \frac{\text{Moment of shear strength along failure arc}}{\text{Moment of weight of failure mass}} \quad (24.7)$$

The denominator is the driving moment and may be evaluated as in Example 24.3. Note that the moment arm for the weight of any slice is equal to $r \sin \theta_i$. Hence we may write

$$M_D = r \sum_{i=1}^n W_i \sin \theta_i$$

where r is the radius of the failure arc, n is the number of slices, and W_i and θ_i are as defined in Fig. 24.11. Similarly, the resisting moment may be written as⁴

$$M_R = r \sum_{i=1}^n (\bar{c} + \bar{\sigma}_i \tan \phi) \Delta l_i = r \left(\bar{c}L + \tan \phi \sum_{i=1}^n R_i \right)$$

where Δl_i is the length of the failure arc cut by the i th slice and L is the length of the entire failure arc. Thus

⁴ The following derivations assume that \bar{c} and ϕ are constant along the failure arc. The equations may be generalized by including \bar{c} and ϕ inside the summations.

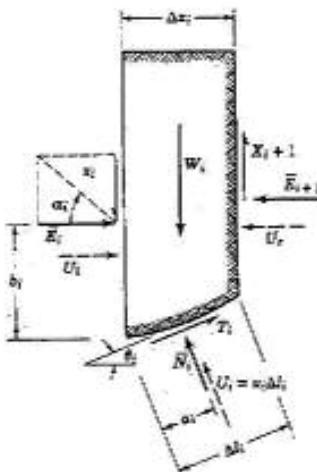


Fig. 24.11 Complete system of forces acting on a slice.

Eq. 24.7 becomes

$$F = \frac{\bar{c}L + \tan \phi \sum_{i=1}^n R_i}{\sum_{i=1}^n W_i \sin \theta_i} \quad (24.8)$$

Equation 24.8 is a perfectly accurate equation. If the R_i used in this equation satisfy statics, then an accurate

Table 24.1 Unknowns and Equations for n Slices

Unknowns Associated with Force Equilibrium

- n Resultant normal forces N_i on the base of each slice or wedge
- 1 Safety factor, which permits the shear forces T_i on the base of each slice to be expressed in terms of N_i
- $n - 1$ Resultant normal forces E_i on each interface between slices or wedges
- $n - 1$ Angles α_i which express the relationships between the shear force X_i and the normal force E_i on each interface
- $3n - 1$ Unknowns, versus $2n$ equations

Unknowns Associated with Moment Equilibrium

- n Coordinates a_i locating the resultant N_i on the base of each wedge or slice
- $n - 1$ Coordinates b_i locating the resultant E_i on each interface between wedges or slices
- $2n - 1$ Unknowns, versus n equations

Total Unknowns

- $5n - 2$ Unknowns, versus $3n$ equations

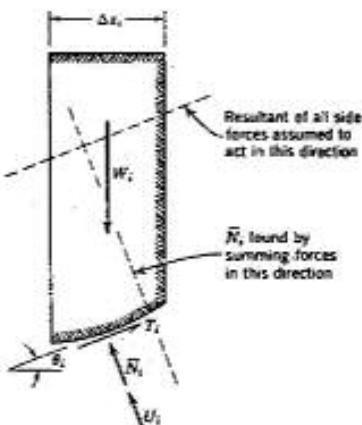


Fig. 24.12 Forces considered in ordinary method of slices.

value of F will result. Moreover, the definition of F in Eq. 24.8 is perfectly consistent with the definition of F in Eqs. 24.4 and 24.5. However, the approximate methods discussed below do not use values of R_i that satisfy statics.

If any external forces other than gravity act on the failure mass (such as the weight of a building upon the slope), the moment of these forces is included in M_D . Pore pressures on the failure arc do not contribute to M_D , since their resultant passes through the center of the arc.

Ordinary Method of Slices

In this method,⁵ it is assumed that the forces acting upon the sides of any slice have zero resultant in the direction normal to the failure arc for that slice. This situation is depicted in Fig. 24.12. With this assumption,

$$R_i + U_i = W_i \cos \theta_i$$

or

$$R_i = W_i \cos \theta_i - U_i = W_i \cos \theta_i - u_i \Delta L_i \quad (24.9)$$

Combining Eqs. 24.8 and 24.9,

$$F = \frac{\delta L + \tan \phi \sum_{i=1}^{n-1} (W_i \cos \theta_i - u_i \Delta L_i)}{\sum_{i=1}^n W_i \sin \theta_i} \quad (24.10)$$

The use of Eq. 24.10 to compute F is illustrated in Example 24.4.

Here the assumption regarding side forces involves $n-1$ assumptions, while there are only $n-2$ unknowns. Hence the system of slices is overdetermined and in general it is not possible to satisfy statics. Thus the safety factor computed by this method will be in error. Numerous examples have shown that the safety factor obtained in this way usually falls below the lower bound of solutions that satisfy statics. In some problems, F from this method may be only 10 to 15% below the range of equally correct answers, but in other problems

⁵ Also known as Swedish Circle Method or Fellenius Method. Consideration of slices within the trial wedge was first proposed by Fellenius (1936).

Example 24.4

Given. Slope in Example 24.3.

Find. Safety factor by ordinary method of slices.

Solution. See Table E24.4.

Table E24.4

Slice	W_i (kN)	$\sin \theta_i$	$W_i \sin \theta_i$ (kN)	$\cos \theta_i$	$W_i \cos \theta_i$ (kN)	u_i (kN/m)	ΔL_i (m)	U_i (kN)	R_i (kN)
1	13.2	-0.03	-0.4	1.00	13.2	0	1.34	0	13.2
2	24.6	0.05	1.2	1.00	24.6	0	0.98	0	24.6
24	19.1	0.14	2.7	0.99	18.9	1.4	0.58	0.8	18.1
3	67.5	0.25	19.6	0.97	65.5	10.0	1.62	16.2	49.3
4	81.8	0.42	34.4	0.91	74.4	13.9	1.71	23.8	50.6
5	84.8	0.58	49.2	0.81	68.7	12.0	1.89	22.7	46.0
6	67.4	0.74	49.9	0.67	45.2	5.3	2.04	10.8	34.4
64	7.2	0.82	5.9	0.57	4.1	0	0.37	0	4.1
7	22.3	0.87	19.4	0.49	10.9	0	2.23	0	10.9
			181.9				12.76		251.2

$$F = \frac{4.31(12.76) + 251.2 \tan 32^\circ}{181.9} = \frac{55.00 + 136.97}{181.9} = \frac{211.97}{181.9} = 1.17$$

Note. That $r \sum W_i \sin \theta_i = 9.15(181.9) = 1664.4$ kNm should equal the moment in the last column of Table E24.3. The slight difference results from rounding errors.

the error may be as much as 60% (e.g., see Whitman and Bailey, 1967).

Despite the errors, this method is widely used in practice because of its early origins, because of its simplicity, and because it errs on the safe side. Hand calculations are feasible, and the method has been programmed for computers. It seems unfortunate that a method which may involve such large errors should be so widely used, and it is to be expected that more accurate methods will see increasing use.

Simplified Bishop Method of Slices

In this newer method^a it is assumed that the forces acting on the sides of any slice have zero resultant in the vertical direction. The forces R_i are found by considering the equilibrium of the forces shown in Fig. 24.13. A value of safety factor must be used to express the shear forces T_i , and it is assumed that this safety factor equals the F defined by Eq. 24.8. Then:

$$R_i = \frac{W_i - u_i \Delta x_i - (1/F) c \Delta x_i \tan \phi}{\cos \theta_i [1 + (\tan \theta_i \tan \phi)/F]} \quad (24.11)$$

Combining Eqs. 24.8 and 24.11 gives

$$F = \frac{\sum_{i=1}^n [\varepsilon \Delta x_i + (W_i - u_i \Delta x_i) \tan \phi] / [1/M_i(\theta)]}{\sum_{i=1}^n W_i \sin \theta_i} \quad (24.12)$$

^a The method was first described by Bishop (1955); the simplified version of the method was developed further by Janbu et al. (1956).

Example 24.5

Given. Slope in Example 24.3.

Find. Safety factor by simplified Bishop method of slices.

Solution. See Table E24.5.

Table E24.5

(1) Slice	(2) Δx_i (m)	(3) $c \Delta x_i$ (kN)	(4) $u_i \Delta x_i$ (kN)	(5) $W_i - u_i \Delta x_i$ (kN)	(6) $(5) \tan \phi$ (kN)	(7) $(3) + (6)$ (kN)	(8) M_i	(9) $(7) / (8)$
							$F = 1.25$	$F = 1.35$
							$F = 1.25$	$F = 1.35$
1	1.37	5.9	0	13.2	8.3	14.2	0.97	0.97
2	0.98	4.2	0	24.6	15.4	19.6	1.02	1.02
2A	0.55	2.4	0.8	18.3	11.4	13.8	1.06	1.05
3	1.52	6.6	15.2	52.3	32.7	39.3	1.09	1.08
4	1.52	6.6	21.1	60.7	37.9	44.5	1.12	1.10
5	1.52	6.6	18.2	66.6	41.6	48.2	1.10	1.08
6	1.34	5.8	7.1	60.3	37.7	43.5	1.05	1.02
6A	0.18	0.8	0	7.2	4.5	5.3	0.98	0.95
7	0.98	4.2	0	22.3	13.9	18.1	0.93	0.92
							232.7	236.4

For assumed

$$F = 1.25 \quad F = \frac{232.7}{181.9} = 1.28$$

$$F = 1.35 \quad F = \frac{236.4}{181.9} = 1.30$$

A trial with assumed $F = 1.29$ would give $F = 1.29$.

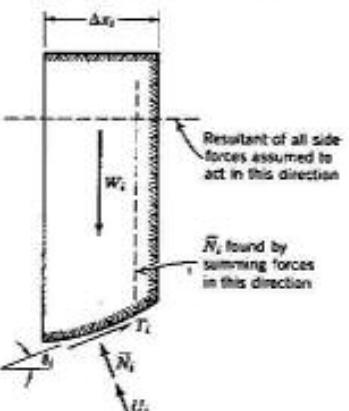
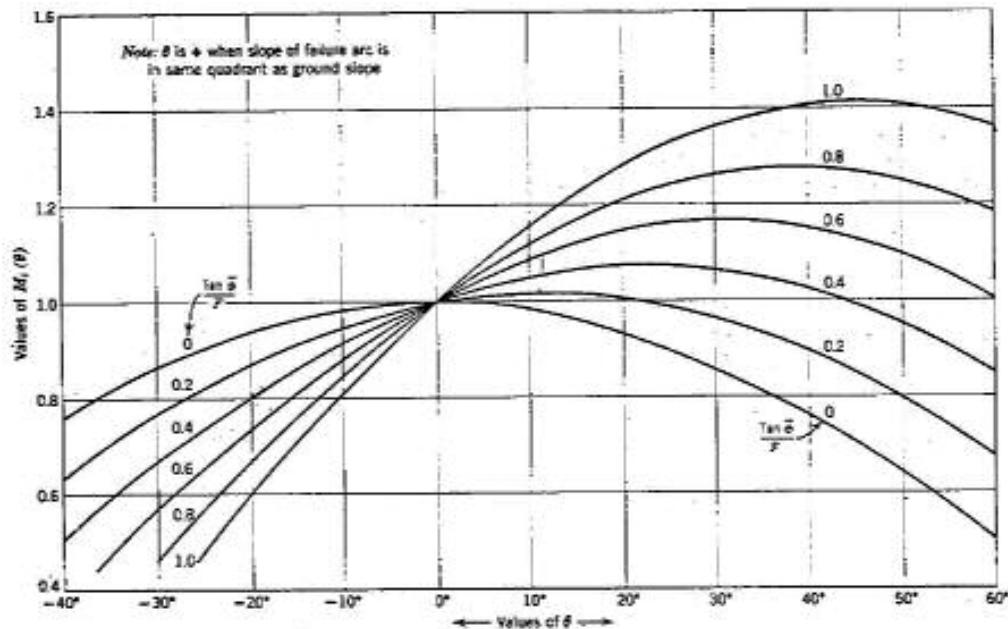


Fig. 24.13 Forces considered in simplified Bishop method of slices.

where

$$M_i(\theta) = \cos \theta_i \left(1 + \frac{\tan \theta_i \tan \phi}{F} \right) \quad (24.13)$$

Equation 24.12 is more cumbersome than Eq. 24.10 from the ordinary method, and requires a trial and error solution since F appears on both sides of the equation. However, convergence of trials is very rapid. Example 24.5 illustrates the tabular procedure which may be used. The chart in Fig. 24.14 can be used to evaluate the function M_i .

Fig. 24.14 Graph for determination of $M_i(\delta)$.

The simplified Bishop method also makes $n - 1$ assumptions regarding unknown forces and hence overdetermines the problem so that in general the values of N_i and F are not exact. However, numerous examples have shown that this method gives values of F which fall within the range of equally correct solutions as determined by exact methods. There are cases where the Bishop method gives misleading results; e.g., with deep failure circles when F is less than unity (see Whitman and Bailey, 1967). Nonetheless, the Bishop method is recommended for general practice. Hand calculations are possible, and computer programs are available.

Other Methods of Slices

There are numerous other versions of this method. In one such method the inclinations α_i of the side forces are assumed (see Lowe and Karafath, 1960; Sherard et al., 1963). Often all α_i are taken equal to the inclination of the slope. This method also overdetermines the system of slices but gives very satisfactory answers. In its present form, this method requires a trial and error graphical solution.

24.6 WEDGE METHOD

In many problems, the potential or actual failure surface can be approximated closely by two or three

straight lines. This situation arises when there are weak strata within or beneath the slope and also when the slope rests upon a very strong stratum. Figures 24.8c and 24.15a illustrate situations where the failure surfaces are almost exactly composed of straight lines. Figure 24.8b shows a situation where use of straight lines gives a very satisfactory approximation. A general version of the method of slices can be used for such problems. However, a satisfactory and usually very accurate estimate of the safety factor can be obtained by the wedge method.

In this method, the potential failure mass is broken up into two or three wedges, as shown in Fig. 24.15b. The shear resistance along the several segments of the failure surface is expressed in terms of the applicable strength parameters and a safety factor F , which is the same for all segments. In Fig. 24.15b there are three unknown forces, (P , N_1 , and N_2), the unknown inclination α of the force between the wedges, and the unknown safety factor. Thus there are five unknowns but only four equations of force equilibrium (two for each wedge), and the system is statically indeterminate. In order to make the system determinate the value of α is assumed. Then the safety factor can be computed.

The wedge method is illustrated in Example 24.6. The strength of the core of the dam is represented by a c with $\phi = 0$. The conditions for which such strength

CONCISE OUTPUT FROM PROGRAM FEA SLOPE

Software Dr. Ing. A. S. Rabuffetti - Milano
PROGRAMMA FEA SLOPE, 2007
ANALISI AGLI ELEMENTI FINITI DEL PENDIO
CRITERIO DI COLLASSO DI COULOMB CON DILATANZA
EFFETTI DI NON LINEARITA' DI TIPO VISCOPLASTICO

DATA DELL'ANALISI 03-10-2009

NUMERO DI PUNTI = 341

NUMERO DI ELEMENTI FINITI PIANI, A 8 NODI = 100

NUMERO DI STRATI DISTINTI DI TERRENO = 2

ANALISI REGIME RISPOSTA DEI TERRENI: SPINTE T RESISTENZE E

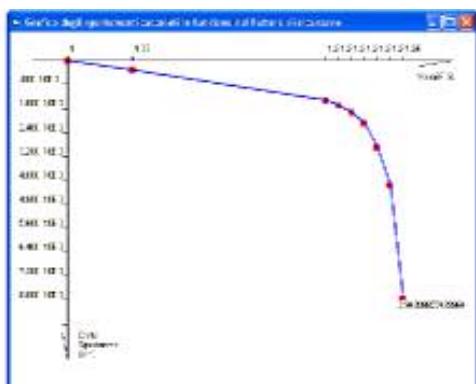
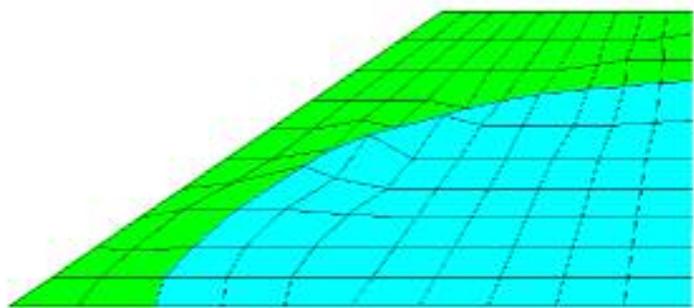
ACCELERAZIONI SISMICHE: a/g H = 0 a/g V = 0

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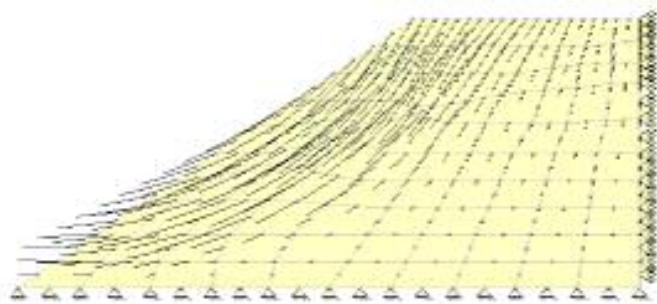
NUMERO DI ITERAZIONI : 0017 0021 0094 0109 0131 0157 0301 0408 0500

DATI GEOTECNICA

SUOLO TIPO	GAMMA TOT. (KN/m ³)	GAMMA EFF. (KN/m ³)	Colore	NSpt Colpi	COES. (KN/m ²)	ATTR. (°)	DILAT. (°)	YOUNG (KN/m ²)	POISSON
1	19.64	19.64	Verde	0	4.31	32	0	25000	.25
2	19.64	9.83	Ciano	0	4.31	32	0	25000	.25



Max Temp - Creation = 214380000 - [Index 10]
 $P_1 = 1.28$
 Required index = 300
 $\text{Growth}_1 P_1 = 1.12333 \cdot 10^{10} = 1.12333$
 Future Total Space = 1.12333e+10



EXAMPLE LAMBE WHITMAN 24.3

EXAMPLE NAVFAC LAYERED SOIL WITH ROCK BASE

GEOTECHNICAL PARAMETERS (SOIL 1)

Natural Unit Weight	γ	=	18.85	KN/m ³
Submerged Unit Weight	γ_1	=	9.04	KN/m ³
Cohesion	C	=	0	KPa
Angle of Internal Friction	ϕ	=	25	°
Dilatancy	δ	=	0	°
Poisson's Ratio	ν	=	0.3	
Soil Modulus (Young's Modulus)	E	=	6.0	MPa

(SOIL 2)

Natural Unit Weight	γ	=	14.46	KN/m ³
Submerged Unit Weight	γ_1	=	4.65	KN/m ³
Cohesion	C	=	28.74	KPa
Angle of Internal Friction	ϕ	=	0	°
Dilatancy	δ	=	0	°
Poisson's Ratio	ν	=	0.5	
Soil Modulus (Young's Modulus)	E	=	5.75	MPa

(ROCK)

Natural Unit Weight	γ	=	24.0	KN/m ³
Submerged Unit Weight	γ_1	=	14.0	KN/m ³
Cohesion	C	=	50	KPa
Angle of Internal Friction	ϕ	=	35	°
Dilatancy	δ	=	0	°
Poisson's Ratio	ν	=	0.25	
Soil Modulus (Young's Modulus)	E	=	500	MPa

REFERENCE (PUBLISHED) SAFETY FACTOR	FEA SLOPE SAFETY FACTOR
1.27	1.27

$$\begin{array}{lcl} \text{FEA SLOPE MAX INTERACTIONS} & = & 1000 \\ \text{FEA SLOPE MAXIMUM ABSOLUTE DEFLECTION} & = & 151 \text{ mm} \end{array}$$

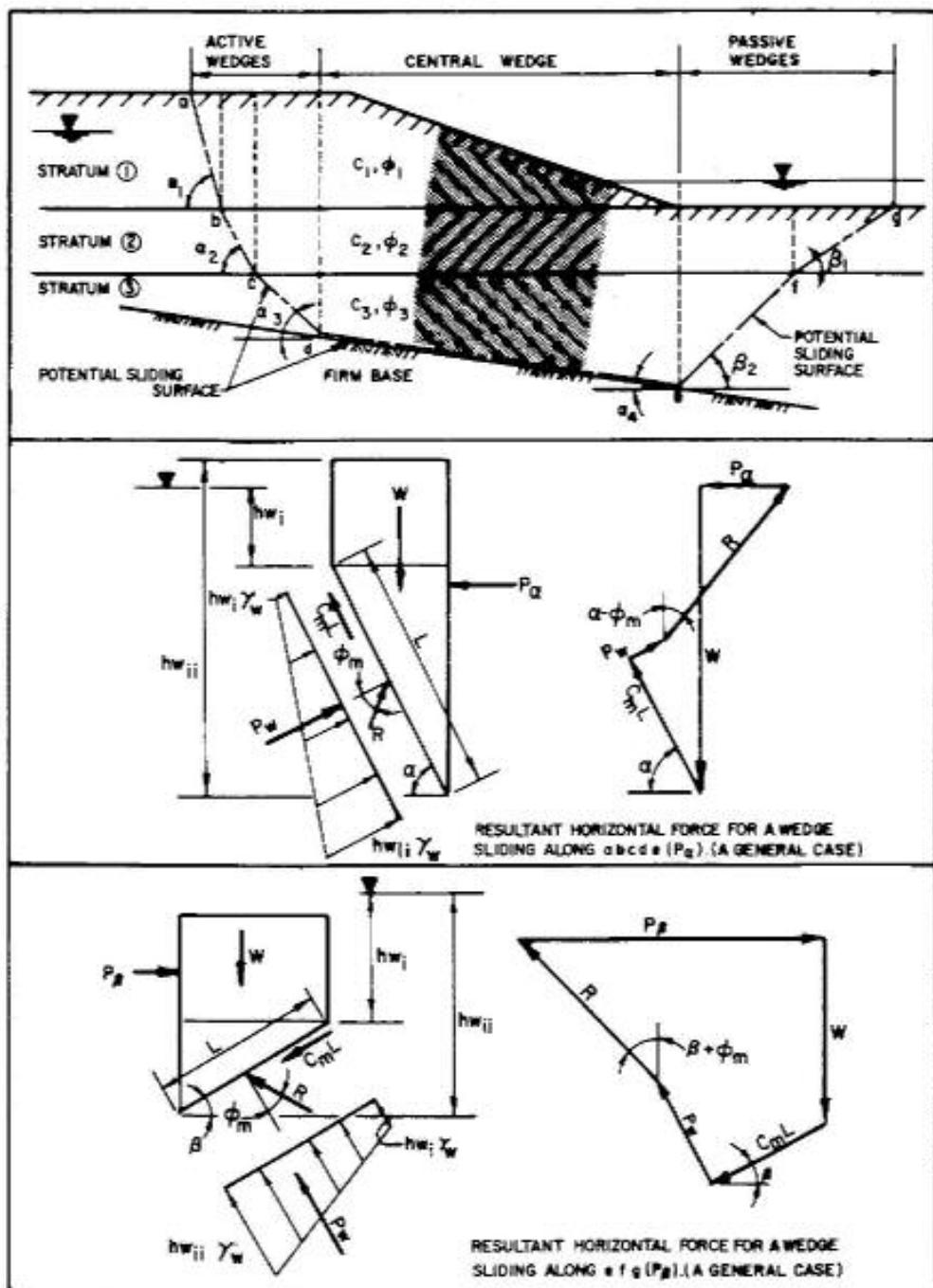


FIGURE 6
Stability Analysis of Translational Failure
7.1-323

DEFINITION OF TERMS

- P_a = RESULTANT HORIZONTAL FORCE FOR AN ACTIVE OR CENTRAL WEDGE ALONG POTENTIAL SLIDING SURFACE α b c d e .
 P_b = RESULTANT HORIZONTAL FORCE FOR A PASSIVE WEDGE ALONG POTENTIAL SLIDING SURFACE β f g .
 W = TOTAL WEIGHT OF SOIL AND WATER IN WEDGE ABOVE POTENTIAL SLIDING SURFACE.
 R = RESULT OF NORMAL AND TANGENTIAL FORCES ON POTENTIAL SLIDING SURFACE CONSIDERING FRICTION ANGLE OF MATERIAL.
 P_w = RESULTANT FORCE DUE TO PORE WATER PRESSURE ON POTENTIAL SLIDING SURFACE CALCULATED AS :
$$P_w = \left[\frac{nw_1 + nw_2}{2} \right] (Lk\gamma_w)$$

 ϕ = FRICTION ANGLE OF LAYER ALONG POTENTIAL SLIDING SURFACE.
 C = COHESION OF LAYER ALONG POTENTIAL SLIDING SURFACE.
 L = LENGTH OF POTENTIAL SLIDING SURFACE ACROSS WEDGE.
 n_w = DEPTH BELOW PHREATIC SURFACE AT BOUNDARY OF WEDGE.
 γ_w = UNIT WEIGHT OF WATER.

PROCEDURES

- 1.) EXCEPT FOR CENTRAL WEDGE WHERE α IS DICTATED BY STRATIGRAPHY USE $\alpha = 45^\circ + \frac{\phi}{2}$, $\beta = 45^\circ - \frac{\phi}{2}$ FOR ESTIMATING FAILURE SURFACE.
- 2.) SOLVE FOR P_a AND P_b FOR EACH WEDGE IN TERMS OF THE SAFETY FACTOR (F_s) USING THE EQUATIONS SHOWN BELOW. THE SAFETY FACTOR IS APPLIED TO SOIL STRENGTH VALUES (TAN ϕ AND C). MOBILIZED STRENGTH PARAMETERS ARE THEREFORE CONSIDERED AS $\phi_m = \tan^{-1}\left(\frac{\tan \phi}{F_s}\right)$ AND $C_m = \frac{C}{F_s}$.

$$P_a = [W - C_m L \sin \alpha - P_w \cos \alpha] \tan [\alpha - \phi_m] - [C_m L \cos \alpha - P_w \sin \alpha]$$

$$P_b = [W + C_m L \sin \beta - P_w \cos \beta] \tan [\beta + \phi_m] + [C_m L \cos \beta + P_w \sin \beta]$$

IN WHICH THE FOLLOWING EXPANSIONS ARE TO BE USED:

$$\tan (\alpha - \phi_m) = \frac{\tan \alpha - \frac{\tan \phi}{F_s}}{1 + \tan \alpha \frac{\tan \phi}{F_s}}$$

$$\tan (\beta + \phi_m) = \frac{\tan \beta + \frac{\tan \phi}{F_s}}{1 - \tan \beta \frac{\tan \phi}{F_s}}$$

- 3.) FOR EQUILIBRIUM $\sum P_a = \sum P_b$, SUM P_a AND P_b FORCES IN TERMS OF F_s , SELECT TRIAL F_s , CALCULATE $\sum P_a$ AND $\sum P_b$. IF $\sum P_a \neq \sum P_b$, REPEAT. PLOT P_a AND P_b VS. F_s WITH SUFFICIENT TRIALS TO ESTABLISH THE POINT OF INTERSECTION (I.E., $\sum P_a = \sum P_b$), WHICH IS THE CORRECT SAFETY FACTOR.
- 4.) DEPENDING ON STRATIGRAPHY AND SOIL STRENGTH, THE CENTER WEDGE MAY ACT TO MAINTAIN OR UPSET EQUILIBRIUM.
- 5.) NOTE THAT FOR $\phi=0$, ABOVE EQUATIONS REDUCE TO :

$$P_a = W \tan \alpha - \frac{C_m L}{\cos \alpha}, \quad P_b = W \tan \beta + \frac{C_m L}{\cos \beta}$$
- 6.) THE SAFETY FACTOR FOR SEVERAL POTENTIAL SLIDING SURFACES MAY HAVE TO BE COMPUTED IN ORDER TO FIND THE MINIMUM SAFETY FACTOR FOR THE GIVEN STRATIGRAPHY.

FIGURE 6 (continued)
Stability Analysis of Translational Failure

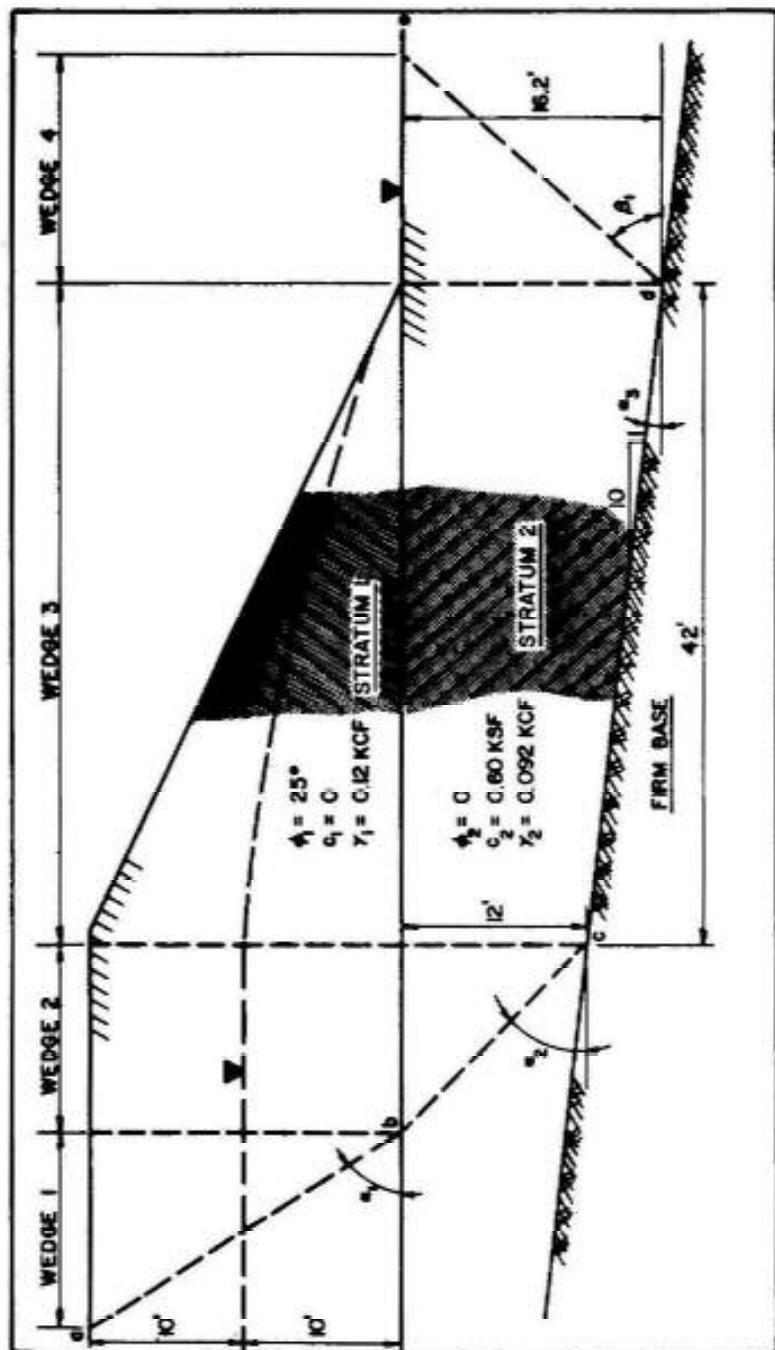


FIGURE 7 Example of Stability Analysis of Translational Failure

FORCES P_a WEDGE 1: $\phi = 25^\circ, C = 0, \gamma = 0.02 \text{ KCF (SLIDING SURFACE ab)}$

$$\alpha_1 = 45 + \phi_1/2 = 57.5^\circ$$

$$W = \frac{20}{2} \times 20 \tan 32.5^\circ \times 0.12 = 15.29 \text{ KIPS}$$

$$P_w = \left(\frac{0+10}{2}\right) (0.062) \times \left(\frac{10}{\sin 57.5^\circ}\right) = 3.68 \text{ KIPS}$$

$$P_{e1} = (W - P_w \cos \alpha_1) \frac{\left(\tan \alpha_1 - \frac{\tan \phi_1}{F_s}\right)}{\left(1 + \tan \alpha_1 \frac{\tan \phi_1}{F_s}\right)} + P_w \sin \alpha_1$$

$$= (15.29 - 1.98) \frac{\left(\frac{1.57 - 0.47}{F_s}\right)}{\left(1 + \frac{0.73}{F_s}\right)} + 3.10 = \left(\frac{20.90 F_s - 6.26}{F_s + 0.73}\right) + 3.10$$

WEDGE 2: $\phi = 0, C = 0.60 \text{ KSF}, \gamma = 0.092 \text{ KCF (SLIDING SURFACE bc)}$

$$\alpha_2 = 45^\circ$$

$$W = 12 \times 10 \times 0.12 + 12 \times 10 \times 0.12 + \frac{12}{2} \times 12 \times 0.092 = 35.42 \text{ KIPS}$$

$$P_{e2} = W \tan \alpha_2 = \frac{F_L}{\cos \alpha_2} \quad (\text{FOR } \phi = 0)$$

$$= 35.42 - \frac{(0.60)(12)}{0.707} = 35.42 - \frac{14.40}{F_s}$$

WEDGE 3: $\phi = 0, C = 0.60 \text{ KSF}, \gamma = 0.092 \text{ KCF (SLIDING SURFACE cd)}$

$$\alpha_3 = \tan^{-1} 0.1 = 5.7^\circ$$

$$W = \frac{20}{2} \times 42 \times 0.12 + \frac{12 + 16.2}{2} \times 42 \times 0.092 = 104.88 \text{ KIPS}$$

$$P_{e3} = W \tan \alpha_3 = \frac{F_L}{\cos \alpha_3} \quad (\text{FOR } \phi = 0)$$

$$= [104.8 \times 0.10] - \left[\frac{\left(0.60 \times \frac{42}{0.99}\right)}{0.99} \right] = 10.48 - \frac{25.71}{F_s}$$

$$\sum P_a = \frac{20.90 F_s - 6.26}{F_s + 0.73} + 49.01 - \frac{49.11}{F_s}$$

FORCES P_b WEDGE 4: $\phi = 0, C = 0.60 \text{ KSF}, \gamma = 0.092 \text{ KCF (SLIDING SURFACE ee)}$

$$\beta_1 = 45^\circ$$

$$W = \frac{16.2}{2} \times 16.2 \times 0.092 = 12.07 \text{ KIPS}$$

$$P_{b1} = W \tan \beta + \frac{F_L}{\cos \beta} \quad (\text{FOR } \phi = 0)$$

$$= 12.07 + \left[\frac{\left(0.60 \times \frac{16.20}{0.707}\right)}{0.707} \right] = 12.07 + \frac{19.44}{F_s}$$

$$\sum P_b = 12.07 + \frac{19.44}{F_s}$$

FIGURE 7 (continued)
Example of Stability Analysis of Translational Failure
7.1-326

SOLVE FOR F_g , FROM $\Sigma P_a = \Sigma P_g$

F_g	ΣP_a	ΣP_g
1.0	17.4	31.5
1.1	21.7	29.7
1.2	25.3	28.3
1.3	28.5	27.0

$$F_g = 1.27$$

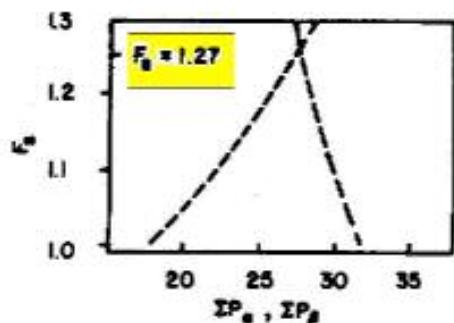


FIGURE 7 (continued)
Example of Stability Analysis of Translational Failure

CONCISE OUTPUT FROM PROGRAM FEA SLOPE

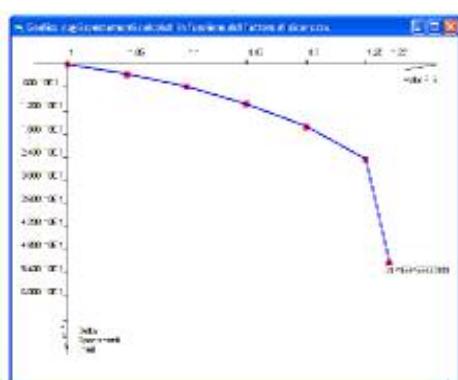
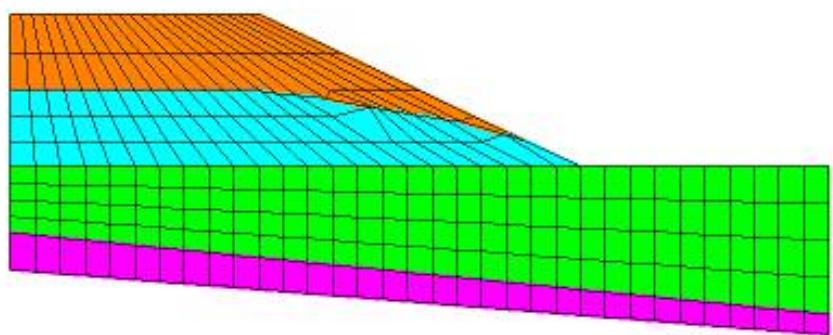
Software Dr. Ing. A. S. Rabuffetti - Milano
PROGRAMMA FEA SLOPE, 2007
ANALISI AGLI ELEMENTI FINITI DEL PENDIO
CRITERIO DI COLLASSO DI COULOMB CON DILATANZA
EFFETTI DI NON LINEARITA' DI TIPO VISCOPLASTICO

DATA DELL'ANALISI 03-10-2009

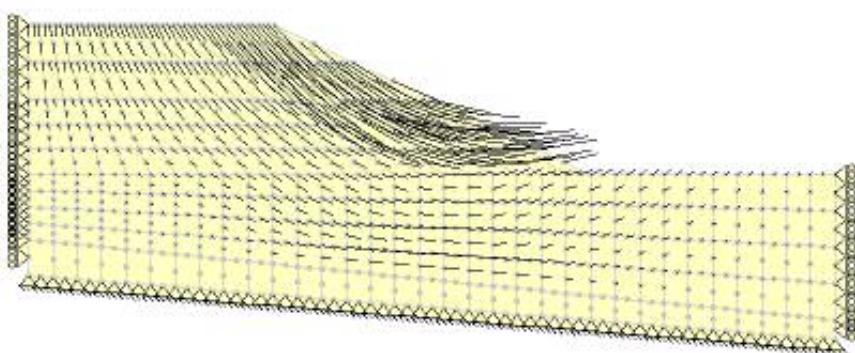
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NUMERO DI ELEMENTI FINITI PIANI, A 8 NODI = 280
NUMERO DI STRATI DISTINTI DI TERRENO = 6
ANALISI REGIME RISPOSTA DEI TERRENI: SPINTE T RESISTENZE E
ACCELERAZIONI SISMICHE: a/g H = 0 a/g V = 0
FATTORI DI SICUREZZA : 1.000 1.050 1.100 1.150 1.200 1.250 1.270
NUMERO DI ITERAZIONI : 0025 0029 0042 0073 0127 0218 1000

DATI GEOTECNICA

SUOLO TIPO	GAMMA TOT.	GAMMA EFF.	Colore	NSpt Colpi	COES. (KN/m ²)	ATTR. (°)	DILAT. (°)	YOUNG (KN/m ²)	POISSON
1	18.85	18.85	Arancio	0	0	25	0	6000	.3
2	18.85	9.04	Ciano	0	0	25	0	6000	.3
3	14.46	4.65	Verde	0	28.74	0	0	5750	.49
4	24	14	Viola	0	50	35	0	500000	.25
5	18.85	18.85	Arancio	0	20	35	0	6000	.3
6	18.85	9.04	Ciano	0	20	35	0	6000	.3



Mass Sped. Assoluto = 150.718337 nni (Nodo 641)
PS = 1.27
Iterazioni di calcolo = 1000.
Sincro alg. Difz. = 0.0000 a/g Vel. = 0.0000
Regime: Force Active = T Resistance = E



EXAMPLE NAVFAC D.M. 7.01 7.1-323

EXAMPLE NAVFAC COHESIVE SOIL

GEOTECHNICAL PARAMETERS

Natural Unit Weight	γ	=	18.07	KN/m ³
Submerged Unit Weight	γ_1	=	8.26	KN/m ³
Cohesion	C	=	28.73	KPa
Angle of Internal Friction	ϕ	=	0	°
Dilatancy	δ	=	0	°
Poisson's Ratio	ν	=	0.5	
Soil Modulus (Young's Modulus)	E	=	5.0	MPa

REFERENCE (PUBLISHED) SAFETY FACTOR	FEA SLOPE SAFETY FACTOR
1.21	1.19

FEA SLOPE MAX INTERACTIONS	=	1000
FEA SLOPE MAXIMUM ABSOLUTE DEFLECTION	=	593 mm

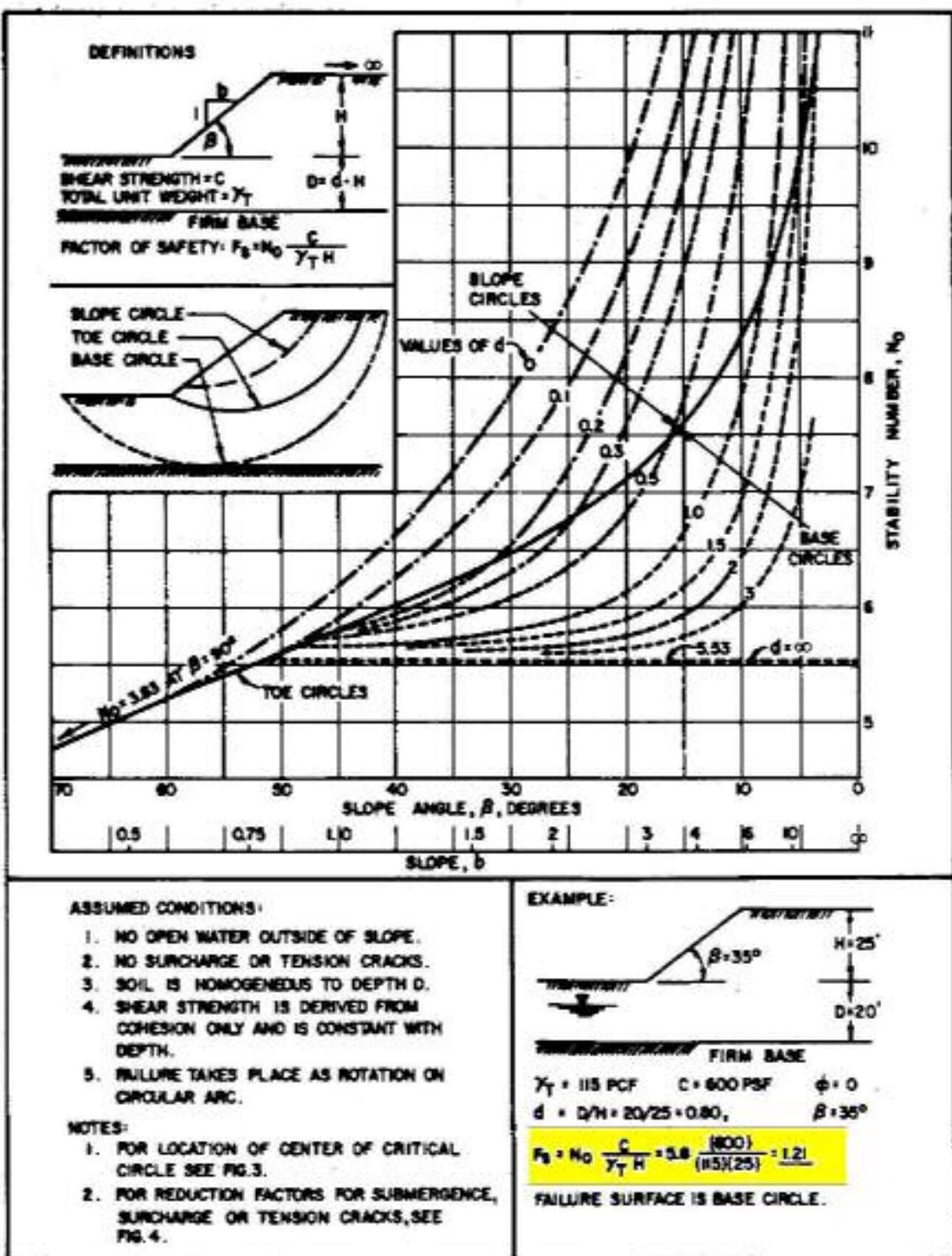


FIGURE 2
Stability Analysis for Slopes in Cohesive Soils, Undrained Conditions,
i.e., Assumed $\phi = 0$

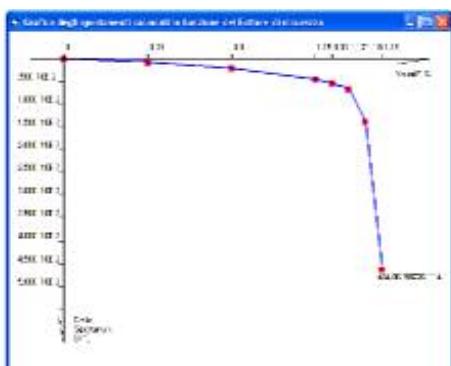
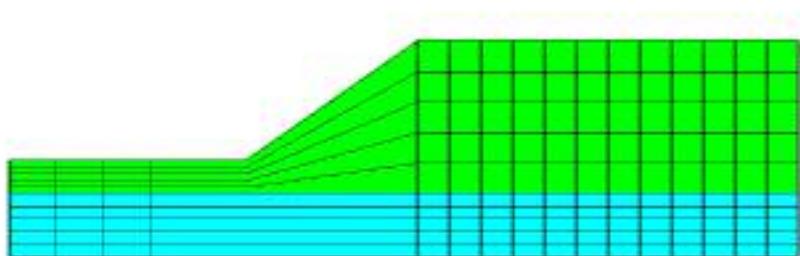
CONCISE OUTPUT FROM PROGRAM FEA SLOPE

Software Dr. Ing. A. S. Rabuffetti - Milano
PROGRAMMA FEA SLOPE, 2007
ANALISI AGLI ELEMENTI FINITI DEL PENDIO
CRITERIO DI COLLASSO DI COULOMB CON DILATANZA
EFFETTI DI NON LINEARITA' DI TIPO VISCOPLASTICO

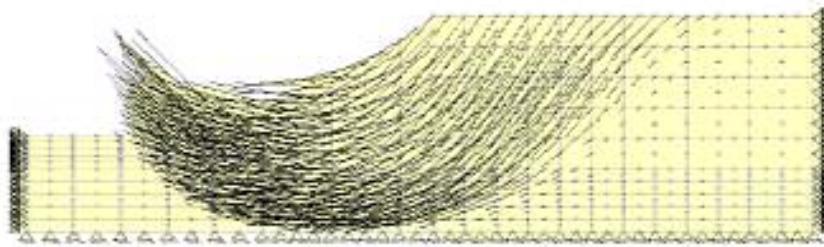
DATA DELL'ANALISI 03-10-2009
NUMERO DI PUNTI = 725
NUMERO DI ELEMENTI FINITI PIANI, A 8 NODI = 220
NUMERO DI STRATI DISTINTI DI TERRENO = 2
ANALISI REGIME RISPOSTA DEI TERRENI: SPINTE T RESISTENZE E
ACCELERAZIONI SISMICHE: a/g H = 0 a/g V = 0
FATTORI DI SICUREZZA : 1.000 1.050 1.100 1.150 1.160 1.170 1.180 1.190
NUMERO DI ITERAZIONI :

DATI GEOTECNICA

SUOLO TIPO	GAMMA TOT.	GAMMA EFF.	Colore	NSpt	COES.	ATTR.	DILAT.	YOUNG	POISSON
	(KN/m ³)	(KN/m ³)		Colpi	(KN/m ²)	(°)	(°)	(KN/m ²)	
1	18.07	8.26	Ciano	0	28.73	0	0	5000	.5
2	18.07	18.07	Verde	0	28.73	0	0	5000	.5



$\Delta x = 120$
 $\Delta z = 120$
 $\Delta h = 120$
Number of Nodes = 3333
Number of Elements = 3333



EXAMPLE NAVFAC D.M. 7.01 7.1-319

EXAMPLE SMITH GRIFFITHS COHESIONLESS SOIL

GEOTECHNICAL PARAMETERS

Unit Weight	γ	=	20.0	KN/m ³
Cohesion	C	=	15	KPa
Angle of Internal Friction	ϕ	=	20	°
Dilatancy	δ	=	0	°
Poisson's Ratio	ν	=	0.3	
Soil Modulus (Young's Modulus)	E	=	100	MPa

REFERENCE (PUBLISHED) SAFETY FACTOR	FEA SLOPE SAFETY FACTOR
1.60	1.60

FEA SLOPE MAX INTERACTIONS = 500
FEA SLOPE MAXIMUM ABSOLUTE DEFLECTION = 40.2 mm

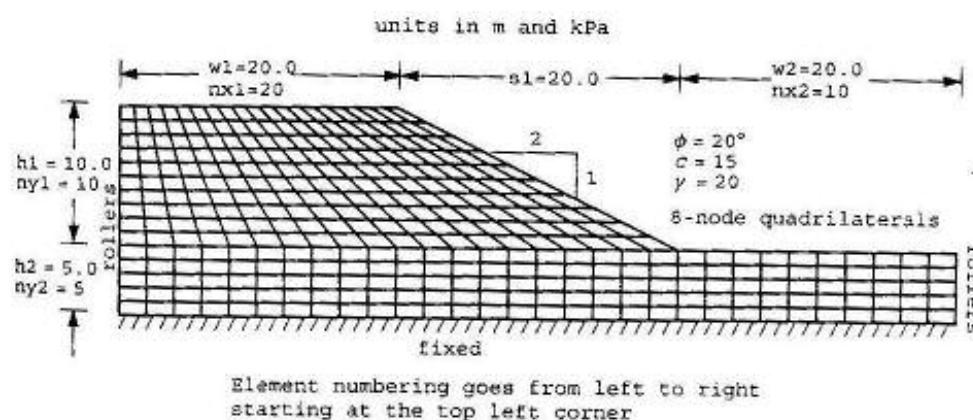
S. G. MAX INTERACTIONS = 500
S. G. MAXIMUM ABSOLUTE DEFLECTION = 37.6 mm

Gravity loads are generated in the manner described in Chapter 5 (5.7) and applied to the slope in a single increment. A trial strength reduction factor loop gradually weakens the soil until the algorithm fails to converge. Each entry of this loop implements a different strength reduction factor SRF . The factored soil strength parameters that go into the elasto-plastic analysis are obtained from,

$$\begin{aligned}\phi_f &= \arctan(\tan \phi / SRF) \\ c_f &= c / SRF\end{aligned}\quad (6.39)$$

Several (usually increasing) values of the SRF factor are attempted until the algorithm fails to converge, at which point SRF is then interpreted as the factor of safety FS . For a detailed description of the algorithm, the reader is referred to Griffiths and Lane (1999).

Subroutines new to this program include `emb_2d_geom` and `emb_2d_bc`. These subroutines generate the mesh and boundary conditions for a standard slope cross-section of the type shown in Figure 6.15, with dimensions and mesh density controlled through the



```
w1     s1     w2     h1     h2
20.0   20.0   20.0   10.0   5.0

nx1   nx2   ny1   ny2
20     10     10     5

np_types
1

prop(phi,c,psi,gamma,e,v)
20.0  15.0  0.0  20.0  1.0e5  0.3

stype(not needed)

tol    limit
0.0001  500

nsrf,(srf(i),i=1,nsrf)
6
1.0  1.2  1.4  1.5  1.55 1.6
```

Figure 6.15 Mesh and data for Program 6.3 example

input data. The boundary conditions are rollers on the left and right vertical boundaries, and full fixity at the base.

Subroutine `mocouf` computes the Mohr-Coulomb failure function F from the current stress state and the factored shear strength parameters (6.12). Subroutine `mocouq` forms the derivatives of the Mohr-Coulomb potential function Q with respect to the three stress invariants and these values are held in `dq1`, `dq2`, and `dq3`. In Programs 6.1 and 6.2, similar subroutines corresponding to the von Mises criterion could have been used, but the required expressions were so trivial that they were written directly into the main program.

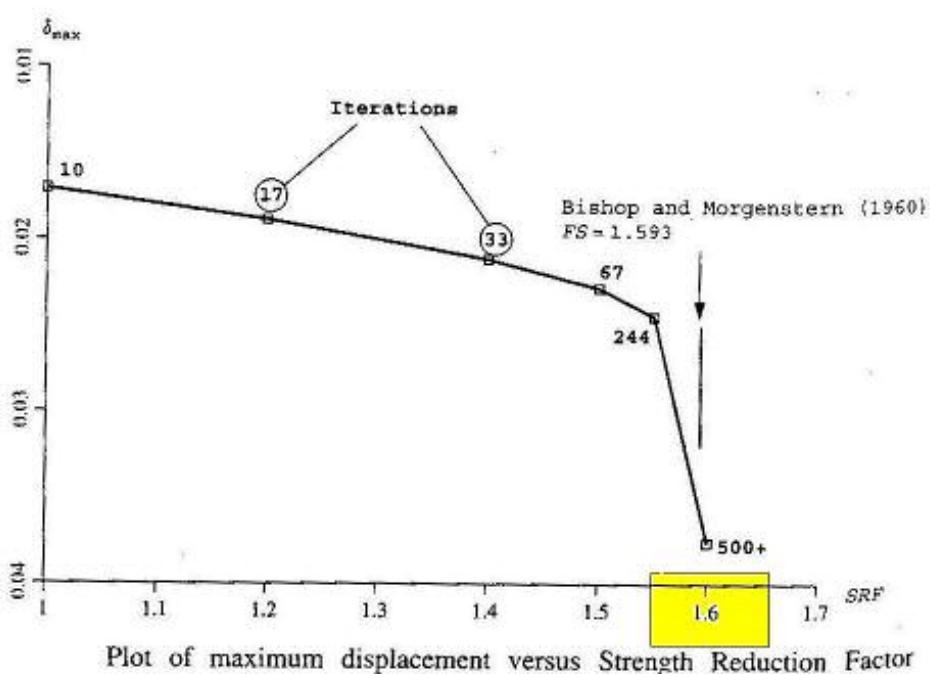
For each material type, six properties must be read in (`nprops=6`): the friction angle ϕ , the cohesion c , the dilation angle ψ , the total unit weight γ , Young's modulus E , and Poisson's ratio ν .

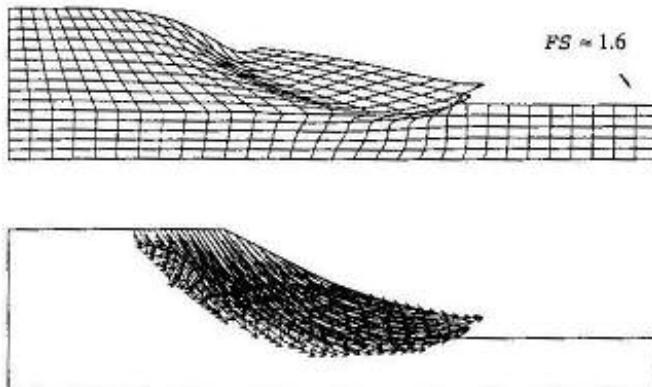
Figure 6.15 shows the mesh and data for an analysis of a homogeneous 2:1 slope with $\phi = 20^\circ$ and $c = 15 \text{ kN/m}^2$. The dilation angle ψ is set to zero and the unit weight is given

There are 2120 equations and the skyline storage is 151000

srf	max disp	iters
1.00	0.1711E-01	10
1.20	0.1889E-01	17
1.40	0.2115E-01	33
1.50	0.2283E-01	67
1.55	0.2446E-01	244
1.60	0.3761E-01	500

Figure 6.16 Results from Program 6.3 example





Deformed mesh and displacement vectors at failure from Program 6.3

as $\gamma = 20 \text{ kN/m}^3$. The elastic parameters are given nominal values of $E = 1 \times 10^5 \text{ kN/m}^2$ and $v = 0.3$ since they have little influence on the computed factor of safety. The convergence tolerance and iteration ceiling are set to `tol=0.0001` and `limit=500` respectively. Six trial strength reduction factors (`nsrf=6`) are input, ranging from 1.0 to 1.6.

No `etype` data is required in this homogeneous example, but if it is required, the user needs to know that element numbering proceeds in the x -direction, starting at the top-left corner of the mesh.

The output in Figure 6.16 gives the strength reduction factor, the maximum nodal displacement at convergence, and the number of iterations to achieve convergence. It can be seen that when `srf=1.6`, the iteration ceiling of 500 was reached. A plot of these results in Figure 6.17 shows that the displacements increase rapidly at this level of strength reduction, indicating a factor of safety of about 1.6. Bishop and Morgenstern's charts (1960) give a factor of safety of 1.593 for the slope under consideration. Figure 6.18 displays the PostScript files `fe95.dis` and `fe95.vec`, which show the deformed mesh and displacement vectors corresponding to slope failure. The mechanism of failure is clearly shown to be of the "toe" type.

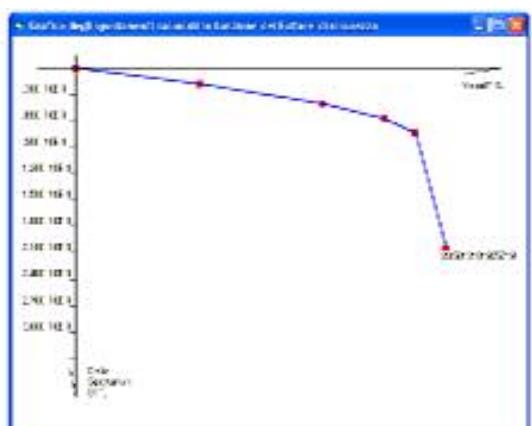
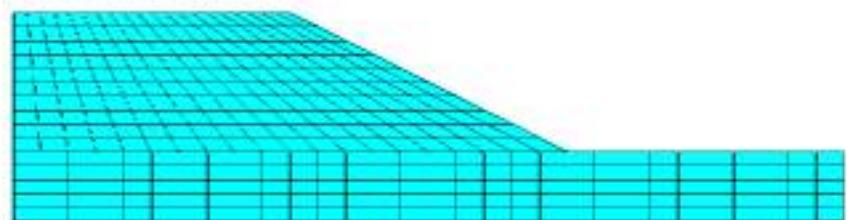
CONCISE OUTPUT FROM PROGRAM FEA SLOPE

Software Dr. Ing. A. S. Rabuffetti - Milano
PROGRAMMA FEA SLOPE, 2007
ANALISI AGLI ELEMENTI FINITI DEL PENDIO
CRITERIO DI COLLASSO DI COULOMB CON DILATANZA
EFFETTI DI NON LINEARITA' DI TIPO VISCOPLASTICO

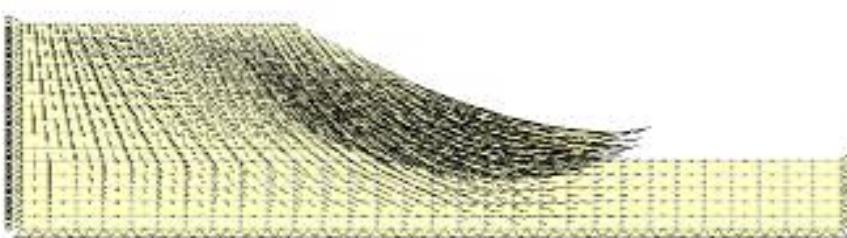
DATA DELL'ANALISI 03-10-2009
NUMERO DI PUNTI = 1141
NUMERO DI ELEMENTI FINITI PIANI, A 8 NODI = 350
NUMERO DI STRATI DISTINTI DI TERRENO = 1
ANALISI REGIME RISPOSTA DEI TERRENI: SPINTE T/E RESISTENZE T/E
ACCELERAZIONI SISMICHE: a/g H = 0 a/g V = 0
FATTORI DI SICUREZZA : 1.000 1.200 1.400 1.500 1.550 1.600
NUMERO DI ITERAZIONI : 0010 0017 0033 0067 0244 0500

DATI GEOTECNICA

SUOLO TIPO	GAMMA TOT. (KN/m ³)	GAMMA EFF. (KN/m ³)	Colore	NSpt Colpi	COES. (KN/m ²)	ATTR. (°)	DILAT. (°)	YOUNG (KN/m ²)	POISSON
1	20	20	Ciano	0	15	20	0	100000	.3



Time Step: Duration=4E-005s (Node 403)
T=0.0
Max stress=100
Min stress=20
Max defl.=11111 mm
Min defl.=11111 mm
Program Run Time = 0.01 Program run time 14



EXAMPLE SMITH – GRIFFITHS PROGRAM 6.3

**EXAMPLE SOIL COHESIONLESS $\phi = 30^\circ$, SLOPE INCLINATION 30°
CONCISE OUTPUT FROM PROGRAM FEA SLOPE**

Software Dr. Ing. A. S. Rabuffetti - Milano
PROGRAMMA FEA SLOPE, 2007
ANALISI AGLI ELEMENTI FINITI DEL PENDIO
CRITERIO DI COLLASSO DI COULOMB CON DILATANZA
EFFETTI DI NON LINEARITA' DI TIPO VISCOPLASTICO

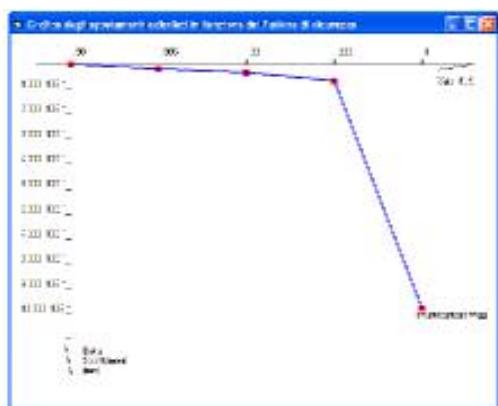
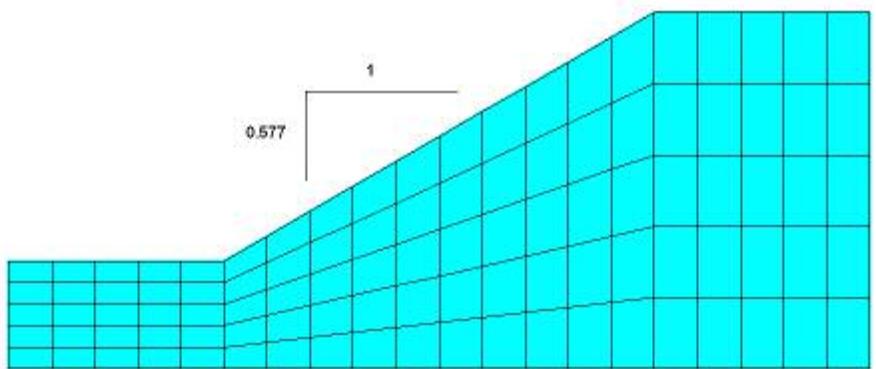
DATA DELL'ANALISI 03-11-2009
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NUMERO DI ELEMENTI FINITI PIANI, A 8 NODI = 100
NUMERO DI STRATI DISTINTI DI TERRENO = 1
ANALISI REGIME RISPOSTA DEI TERRENI: SPINTE T/E RESISTENZE T/E
ACCELERAZIONI SISMICHE: a/g H = 0 a/g V = 0
FATTORI DI SICUREZZA : 0.980 0.985 0.990 0.995 1.000
NUMERO DI ITERAZIONI : 0055 0060 0088 0127 1000

DATI GEOTECNICA

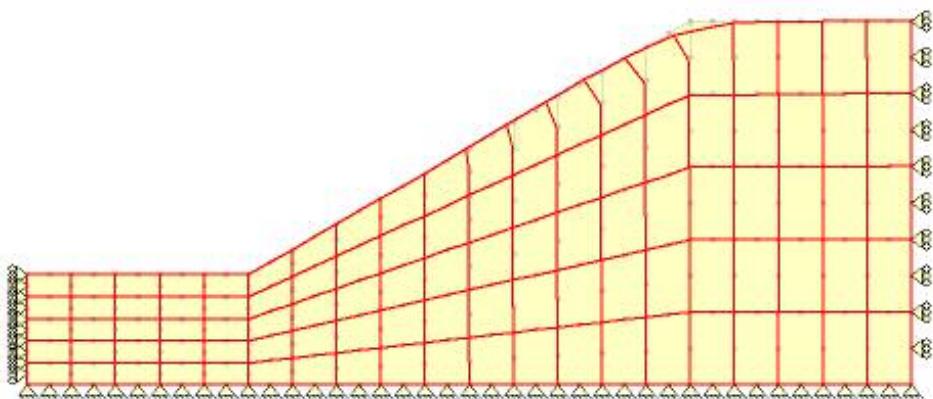
SUOLO TIPO	GAMMA TOT.	GAMMA EFF.	Colore	NSpt Colpi	COES. (KN/m ²)	ATTR. (°)	DILAT. (°)	YOUNG (KN/m ²)	POISSON
1	17	0	Ciano	0	0	30	0	20000	.3

**FEA SLOPE
SAFETY FACTOR**

1.000



Mai Spur = 2352.445587 mm
 PS = 1.
 Iterazioni di calcolo = 1000.
 Sincro alg Distr = 0.0000 a/g Vert. = 0.0000
 Pagine: Force Active = T/E Persistence = T/E



**EXAMPLE SOIL COHESIONLESS $\phi = 35^\circ$, SLOPE INCLINATION 35°
CONCISE OUTPUT FROM PROGRAM FEA SLOPE**

Software Dr. Ing. A. S. Rabuffetti - Milano
PROGRAMMA FEA SLOPE, 2007
ANALISI AGLI ELEMENTI FINITI DEL PENDIO
CRITERIO DI COLLASSO DI COULOMB CON DILATANZA
EFFETTI DI NON LINEARITA' DI TIPO VISCOPLASTICO

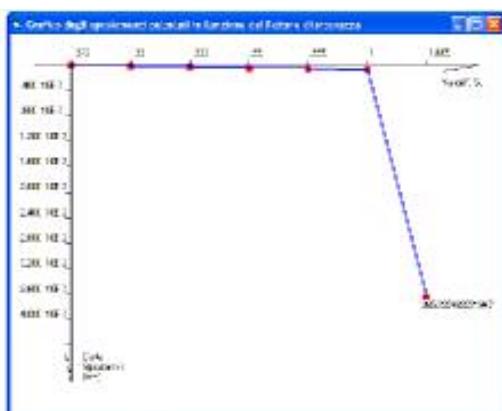
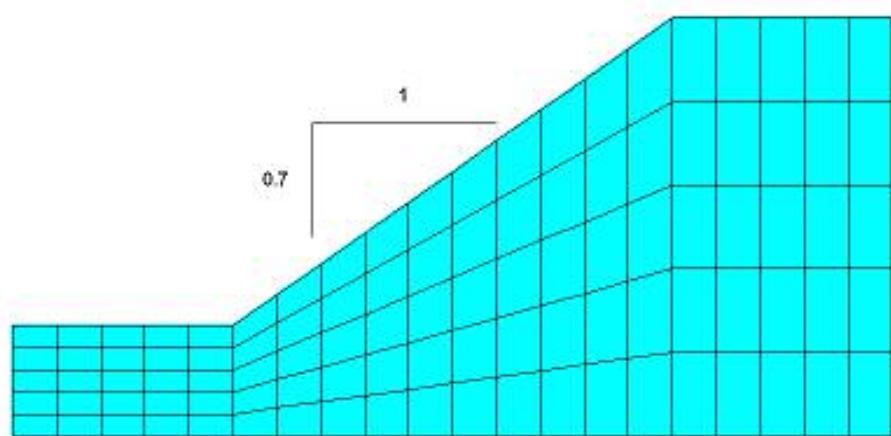
DATA DELL'ANALISI 03-11-2009
NUMERO DI PUNTI = 351
NUMERO DI ELEMENTI FINITI PIANI, A 8 NODI = 100
NUMERO DI STRATI DISTINTI DI TERRENO = 1
ANALISI REGIME RISPOSTA DEI TERRENI: SPINTE T/E RESISTENZE T/E
ACCELERAZIONI SISMICHE: a/g H = 0 a/g V = 0
FATTORI DI SICUREZZA : 0.975 0.980 0.985 0.990 0.995 1.000 1.005
NUMERO DI ITERAZIONI : 0050 0052 0055 0057 0075 0108 1000

DATI GEOTECNICA

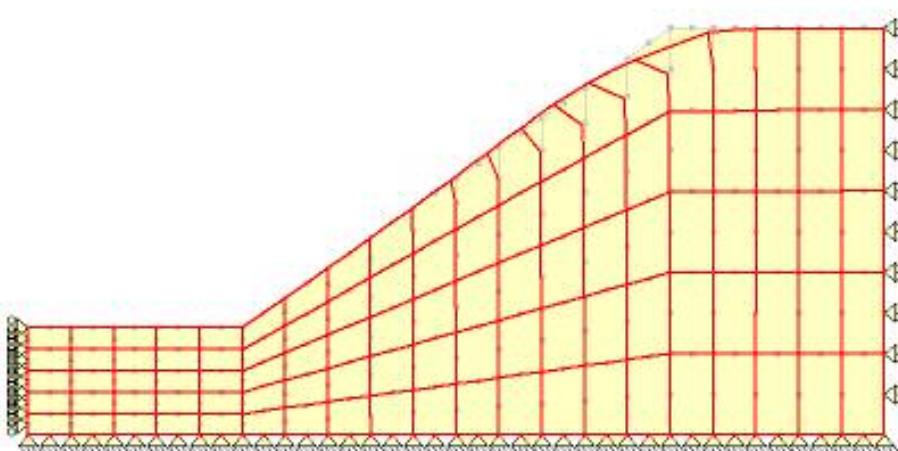
SUOLO TIPO	GAMMA TOT.	GAMMA EFF.	Colore	NSpt Colpi	COES. (KN/m ²)	ATTR. (°)	DILAT. (°)	YOUNG (KN/m ²)	POISSON
1	18	0	Ciano	0	0	35	0	30000	.275

**FEA SLOPE
SAFETY FACTOR**

1.005



Max.Spost = 2585.646477 mm
 FS = 1.005
 Iterazioni di calcolo = 1000.
 Simm a/g Oriz. = 0.0000 a/g Vert. = 0.0000
 Regime: Freno Attivo = T/E Resistenza = T/E



**EXAMPLE SOIL COHESIONLESS $\phi = 40^\circ$, SLOPE INCLINATION 40°
CONCISE OUTPUT FROM PROGRAM FEA SLOPE**

Software Dr. Ing. A. S. Rabuffetti - Milano
PROGRAMMA FEA SLOPE, 2007
ANALISI AGLI ELEMENTI FINITI DEL PENDIO
CRITERIO DI COLLASSO DI COULOMB CON DILATANZA
EFFETTI DI NON LINEARITA' DI TIPO VISCOPLASTICO

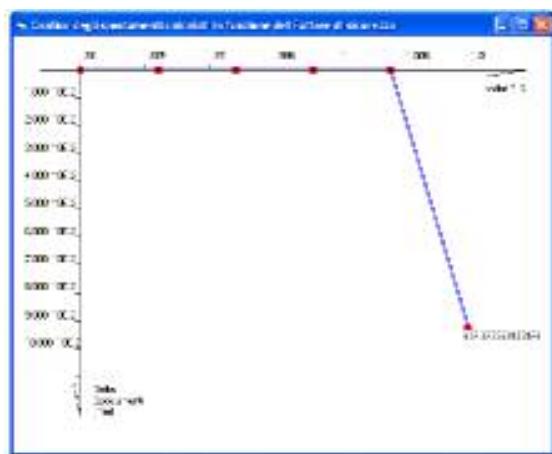
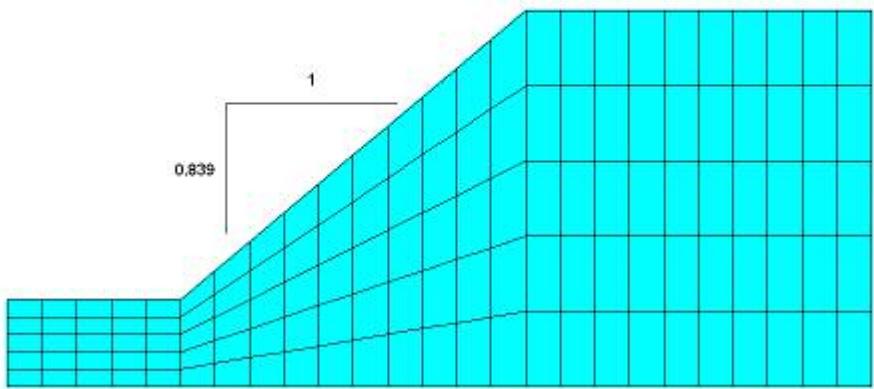
DATA DELL'ANALISI 03-11-2009
NUMERO DI PUNTI = 436
NUMERO DI ELEMENTI FINITI PIANI, A 8 NODI = 125
NUMERO DI STRATI DISTINTI DI TERRENO = 1
ANALISI REGIME RISPOSTA DEI TERRENI: SPINTE T/E RESISTENZE T/E
ACCELERAZIONI SISMICHE: a/g H = 0 a/g V = 0
FATTORI DI SICUREZZA : 0.980 0.985 0.990 0.995 1.000 1.005
NUMERO DI ITERAZIONI : 0057 0058 0059 0061 0065 1000

DATI GEOTECNICA

SUOLO TIPO	GAMMA TOT. (KN/m ³)	GAMMA EFF. (KN/m ³)	Colore	NSpt Colpi	COES. (KN/m ²)	ATTR. (°)	DILAT. (°)	YOUNG (KN/m ²)	POISSON
1	20	0	Ciano	0	0	40	0	50000	.24

**FEA SLOPE
SAFETY FACTOR**

1.005



Max Spolt = 3188.15975 inn
 FS = 1.005
 Iterazioni di calcolo = 1000
 Sfondi rig. Orizz. = 0.0000 rig. Vert. = 0.0000
 Regime: Fosse Attive = T/E Resistenza = T/E

